An adaptive diagnostic algorithm for a distributed system modeled by dual-cubes *

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Abstract

Problem diagnosis in large distributed computer systems and networks is a challenging task that requires fast and accurate inferences from huge data volumes. In this paper, the PMC diagnostic model are considered and the diagnosis approach in this model is based on end-to-end probing technology. A probe is a test transaction whose outcome depends on some of the system’s component; diagnosis is performed by appropriately selecting the probes and analyzing the results. In the PMC mode, every computer can execute a probe to test dedicated system’s components. The key point of the PMC model is that any test result reported by a faulty probe station is unreliable and the test result reported by fault-free probe station is always correct. The aim of the diagnosis is to locate all faulty components in the system based on the collection of the test results. The fault diagnosis problem in an unstructured network has been shown to be NP-hard. We address an special structured network, namely dual cubes, in this paper. An \( (n+1) \)-connected dual-cube \( DC(n) \) is an \( (n+1) \)-regular spanning subgraph of a \( (2n+1) \)-dimensional hypercube. It uses \( n \)-dimensional hypercubes as building blocks and keeps the main desired properties of the hypercube so that it is suitable to be used as a topology of distributed systems. In this paper, we prove the dual cube is adaptively diagnosed using at most 3 parallel testing rounds, with at most \( n+1 \) faulty nodes, where each node participates in at most one test in each round. Furthermore, we propose an adaptive diagnostic algorithm for the \( DC(n) \) and show that it is adaptively diagnosed with at most 3 parallel testing rounds and at most \( N + O(n^3) \) tests, where \( N = 2^{2n+1} \) is the size of the \( DC(n) \).

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1 Introduction

Accurate diagnosis of unobserved states of a large multi-component distributed system based on the results of various tests and measurements is a common problem occurring in practice. For example, in real-time monitoring, diagnosis of the “health” of a large cluster system containing hundreds or thousands of workstations performing distributed computations. In order to achieve high diagnostic accuracy, a large number of tests should be required performing that can be quite expensive. It is therefore essential to improve cost-efficiency of diagnosis by using only the most relevant measurements to the system.

A graph-theoretical model of fault diagnosis termed *PMC model* was proposed in a classic paper by Preparata, Metze and Chien [10]. In this model, each component in the system is either faulty or fault-free. The fault-status of a component does not change during the diagnosis. The diagnosis approach in this model is based on end-to-end probing technology. A probe is a test transaction whose outcome depends on some of the system’s component; diagnosis is performed by appropriately selecting the probes and analyzing the results. In the PMC mode, every computer can execute a probe to test dedicated system’s components. The key point of PMC model is that any test result reported by a faulty probe station is unreliable and the test result reported by fault-free probe station is always correct. The aim of the diagnosis is to locate all faulty components in the system based on the collection of the test results.

The topology of a distributed system is conveniently represented with an undirected graph in which the vertices (edges) of the graph represent the processors (communication links) of the network. The fault diagnosis problem in an unstructured network has been shown to be NP-hard. The early researchers in this area focused on *one-step* or *non-adaptive* diagnosis in structured networks. In this type of diagnosis, all test assignments are determined in advance and they can not be rescheduled during the diagnosis process. Nakajima [8] has
first proposed an *adaptive diagnosis* approach that the next tests can be rescheduled after seeing the results of previous ones. An adaptive diagnosis proceeds in several test rounds, and the objective of the problem is to reduce the number of rounds as well as the total number of tests.

Kranakis et al. [4] showed adaptive diagnosis algorithms using minimum number of tests for trees, cycles, and tori. Okashita et al. [9] proposed adaptive diagnosis algorithms using optimal number of tests and constant testing rounds for wrapped butterfly networks. For $n$-dimensional hypercube with at most $t = n$ node faults, Feng et al. [2] presented an algorithm using at most $2^n \times ([\log t] + 2)$ tests in at most $t + 4$. Kranakis and Pelc [5] improved their work and showed that $2^n + 3t/2$ tests are sufficient, and proposed a diagnosis scheme being performed in at most 11 rounds. Further, Bjorklund [1] proved that the optimal number $2^n + t - 1$ of tests is sufficient and that diagnosis can be performed in 4 rounds. Fujita and Araki [3] proposed a optimal scheme that completes a diagnosis for the $n$-dimensional hypercube in at most three rounds.

This paper deals with the dual-cube proposed in [12, 14, 15] using low-dimensional hypercubes as building blocks, called clusters, and preserving the main desired properties of the hypercube. The $n$-dimensional dual-cube termed $DC(n)$ is an $(n + 1)$-regular spanning subgraph of a $(2n + 1)$-dimensional hypercube; thus, it greatly increases the total number of nodes in the system compared with the hypercube of the same node degree. We first show that the diagnosability of $DC(n)$ is $n + 1$ and then show that adaptive diagnosis is possible using at most $N + n$ tests for an $N$-nodes distributed system modeled by dual-cubes $DC(n)$ in which at most $n + 1$ processes are faulty, where $N = 2^{2n+1}$ and $n \geq 1$.

The outline of the paper is as follows. Section 2 provides the basic definition and notation. Section 3 gives an algorithm to constructed a virtual Hamiltonian cycle in $DC(n)$. The main adaptive diagnostic algorithm is proposed and proved in Section 4. Section 5 provides a
2 Preliminaries

The topology of a distributed system is conveniently represented with an undirected graph in which the vertices (edges) of the graph represent the components (communication links) of the network. Let $V$ be a set of system components or nodes that each node in $V$ can be either “fault-free” (functioning correctly) or “faulty” (functioning incorrectly). The nodes may be physical entities such as routers, severs, and links, or logical entities such as software components, database tables, etc. Throughout this paper, the terms vertex and node, edge and link, and graph and network are used interchangeably. For the graph definitions and notations we follow [7]. $G = (V, E)$ is a graph if $V$ is a finite set and $E$ is a subset of $\{ (u, v) \mid (u, v) \text{ is an unordered pair of } V \}$. The set of all vertices of $G$ is denoted by $V(G)$ and the set of all edges of $G$ is denoted by $E(G)$. A graph $H$ is a subgraph of a graph $G$ if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$.

A probe or test is a method of obtaining information about the system components. A test of node $u$ to another node $v$, denoted $(u, v)$, is possible if these two nodes are joined in $G$. An outcome of the test $(u, v)$ is denoted by $\sigma(u, v)$; thus, $\sigma(u, v) = 0$ if $u$ evaluates $v$ as fault-free and $\sigma(u, v) = 0$ if $u$ judges $v$ is faulty. The outcome $\sigma(u, v)$ of test $(u, v)$ is always correct if $u$ is fault-free, but if $u$ is faulty, the result is unreliable. The collection of all test results is called a syndrome. In this paper, we assume a diagnosis process proceeds in several rounds and in each round, each vertex can participate at most one test, i.e., if two tests $(u, v)$ and $(x, y)$ are conducted in the same round, four vertices $u, v, x, y$ must be distinct. Throughout this paper, a test with result 0 (resp. 1) is referred to as 0-arrow (resp. 1-arrow).

The $n$-dimensional hypercube $Q_n$ contains $2^n$ vertices and has $n$ neighbors per node.
The address of vertex is assigned by unique $n$-bit binary string written as $u = u_{n-1} \ldots u_0$ where $u_i \in \{0,1\}$ and $0 \leq i \leq n - 1$. Two vertices are adjacent if and only if these two node addresses differ in exactly one bit. The Hamming distance $h(u, v)$ between two vertices $u$ and $v$ is the number of different bits in the corresponding strings of both vertices. This paper deals with a dual-cube that uses hypercubes as basic components and each hypercube component is referred to as a cluster. In an $n$-dimensional dual-cube $DC(n)$, there are two classes with each class consisting of $2^n$ clusters. Therefore, the node address is labeled with a unique $(2n + 1)$-bit binary string. The leftmost bit indicates the type of class (class 0 and class 1). Each node in a $DC(n)$ has degree $n + 1$ where $n$ links are used within a cluster and a single link is used to connect a node to a cluster of the other class. However, there are no links between the clusters of the same class. Note, a $DC(n)$ is a spanning subgraph of a $(2n + 1)$-dimensional hypercube $Q_{2n+1}$. Formally, $DC(n)$ can be defined as follows.

**Definition 1** A dual-cube $DC(n)$ consists of $2^{2n+1}$ vertices, and each vertex is labeled with a unique $(2n + 1)$-bit binary string and has $n + 1$ neighbors. There is an edge between two nodes $u = u_{2n}u_{2n-1} \ldots u_0$ and $v = v_{2n}v_{2n-1} \ldots v_0$ if and only if the following conditions are satisfied:

1. $u$ and $v$ differ exactly in one bit position $i$,
2. if $0 \leq i \leq n - 1$ then $u_{2n} = v_{2n} = 0$ and
3. if $n \leq i \leq 2n - 1$ then $u_{2n} = v_{2n} = 1$.

The set of vertices $u$ of form $0u_{2n-1} \ldots u_n \,*\,*\,*\,*$, where * means “don’t care” constitutes an $n$-dimensional hypercube. We call these hypercubes clusters of class 0. Similarly, the set of vertices $u$ of form $1\,*\,*\,*\,*u_{n-1} \ldots u_0$ constitutes an $n$-dimensional hypercube and we call them clusters of class 1. In addition, an edge connecting two nodes in two clusters of distinct
classes is termed a cross-edge. In other words, \( e = (u, v) \) is a cross-edge if and only if \( u \) and \( v \) differ in the leftmost bit position. Moreover, each node in a \( DC(n) \) is identified by a unique \((2n+1)\)-bit binary string, an id. Each id contains three parts: 1-bit \( \text{class} \_ \text{id} \), \( n \)-bit \( \text{cluster} \_ \text{id} \) and \( n \)-bit \( \text{node} \_ \text{id} \). In the following discussion, we use \( id = (\text{class} \_ \text{id}, \text{cluster} \_ \text{id}, \text{node} \_ \text{id}) \) to denote the node address. The bit-position of \( \text{cluster} \_ \text{id} \) and \( \text{node} \_ \text{id} \) depends on the value of \( \text{class} \_ \text{id} \). If \( \text{class} \_ \text{id} = 0 \) (\( \text{class} \_ \text{id} = 1 \)), then \( \text{node} \_ \text{id} \) is the rightmost \( n \) bits and \( \text{cluster} \_ \text{id} \) is the next \( n \) bits. The cluster containing vertex \( u \) is denoted as \( C_u \). For any two nodes \( u \) and \( v \) in a \( DC(n) \), \( C_u = C_v \) if and only if \( u \) and \( v \) are in the same cluster. For example, Figure 1 shows a \( DC(2) \).

Under the PMC model, a self-diagnosable system \( G \) was often modeled by a digraph in which an arc direct from vertex \( u \) to vertex \( v \) means that \( u \) can test \( v \). In this situation, \( u \) is a tester and \( v \) is a tested vertex. We use a graph \( G = (V, E) \) to represent a self-diagnosable system. For a vertex \( u \) of \( G \), the set of all its neighboring vertices is termed by \( N(u) \), i.e., \( N(u) = \{v \in V \mid (u, v) \in E\} \). For a subset \( V' \subseteq V \), let \( N(V') = \bigcup_{v \in V'} N(v) \).

**Definition 2** Under the PMC model, a syndrome \( \sigma \) for system \( G \) is defined as follows: For any two adjacent vertices \( u \) and \( v \),

\[
\sigma(u, v) = \begin{cases} 
0, & \text{if } v \text{ is tested by } u \text{ to be fault-free;} \\
1, & \text{if } v \text{ is tested by } u \text{ to be faulty.}
\end{cases}
\]
For each subset \( F \subseteq V \), let \( \sigma(F) \) denote the set of all possible syndromes that can be produced if \( F \) is the set of all faulty nodes. For a given syndrome \( \sigma \), a fault set \( F \subseteq V \) is called to be consistent with \( \sigma \) if the following two conditions are satisfied:

1. \( v \in V - F \) if \( u \in V - F \) and \( \sigma(u, v) = 0 \);
2. \( v \in F \) if \( u \in V - F \) and \( \sigma(u, v) = 1 \).

We have the following definition of diagnosability.

**Definition 3** A system is said to be \( t \)-diagnosable if for any syndrome \( \sigma \), there is at most one faulty subset \( F \subseteq V \) that is consistent with \( \sigma \), given that \( |F| \leq t \).

Lai et al. [6] give a sufficient and necessary condition to characterize \( t \)-diagnosable systems.

**Lemma 1** [6] A system \( G \) is \( t \)-diagnosable if and only if for each \( S \subset V(G) \) with \( |S| = p \), \( 0 \leq p \leq t - 1 \), every connected component of \( G - S \) has at least \( 2(t - p) + 1 \) vertices.

It is a well known fact that the dual-cubes \( DC(n) \) has connectivity \( n + 1 \). Therefore, the following lemma is obtained.

**Lemma 2** For \( n \geq 1 \), the dual-cube \( DC(n) \) is \( (n + 1) \)-diagnosable.

Given a syndrome \( \sigma \) on a graph \( G \): A \( \sigma \)-zero cycle within \( G \) is a cycle on which the outcomes of the tests are all 0. All nodes on a \( \sigma \)-zero cycle are \( \sigma \)-zero nodes on the cycle. A \( \sigma \)-nonzero cycle within \( G \) is a cycle on which the outcome of at least one test is 1. The following result follows from the assumptions of the PMC model.

**Lemma 3** For \( n \geq 1 \), let \( C \) be a \( k \)-cycle within a \( DC(n) \) containing at most \( n + 1 \) faulty nodes, and \( \sigma \) is a syndrome on \( DC(n) \).
(1) If $C$ is $\sigma$-zero and $k > n + 1$, then all nodes on $C$ are fault-free.

(2) If $C$ is $\sigma$-nonzero, then at least one node on $C$ is faulty.

3 Disjoint virtual Hamiltonian cycles

To develop an adaptive diagnosis algorithm for $DC(n)$ running at most three testing rounds, we should find $2^{n-1}$ disjoint cycles with equal size in $DC(n)$ so each cycle contains $2^{n+2}$ vertices and exactly one cube-edge in each cluster is contained in this cycle. We call such cycle a virtual Hamiltonian cycle due to traversing a cube-edge of each cluster exactly once (see Figure 2). For any given an edge $e$, let $VHC_e$ denote a virtual Hamiltonian cycle passing through $e$.

![Figure 2: A virtual Hamiltonian cycle of a dual-cube.](image)

In [13], a construction schema of a virtual Hamiltonian cycle was proposed using an extended double-dimension list, $EDD(n)$, defined as follows. Let the reflected double-dimension list be $DDL(n) = (DDL(n-1), n-1, n-1, DDL(n-1))$ if $n > 1$, and $DDL(1) = (0, 0)$. Then the extended double-dimensional list $EDD(n) = (DDL(n), n-1, n-1)$. Since there are
two classes in a dual-cube, $EDD(n)$ doubles each dimension number in an extended list consisting of $D(n)$ plus the highest dimension $n−1$. For example, $EDD(2) = (0, 0, 1, 0, 1, 1)$. Then a virtual Hamiltonian cycle can be generated with $EDD(n)$ and any starting vertex $u$.

Algorithm 1 dualCubeVHC($n, u$)

1: $EDD(n) = (DDL(n), n−1, n−1)$;
2: for each dimension number $i$ in $EDD(n)$ do
3:   if ($u$ is of class 0) then
4:     $v = u \oplus 2^i$;
5:   else
6:     $v = u \oplus 2^{n+i}$;
7:   end if
8:   $P = P \cup \{(u, v)\}$;
9:   $u = v \oplus 2^n$;
10: end for

![Figure 3: Two disjoint virtual Hamiltonian cycles in a DC(2).](image)

Given a starting vertex $u$ of $DC(n)$ and an integer $0 \leq d \leq 2n − 1$, a virtual Hamiltonian cycle containing $(u, v)$, $u = v \oplus 2^d$, can be generated by algorithm dualCubeVHC. For instance, a virtual Hamiltonian cycle $VHC_{e_0}$ and $VHC_{e_1}$ of $DC(2)$ starting vertex 00000 and 00011, respectively, can be generated by dualCubeVHC(2,00000) and dualCubeVHC(2,00011) (see Figure 3). In [11], it has proved that a $DC(n)$ can be decomposed
by $2^{n-1}$ disjoint virtual Hamiltonian cycles with $2^{n-1}$ distinct vertices where $h(u, v)$ is even for any pair vertices $u$ and $v$ of them.

**Lemma 4** [11] For $n \geq 2$, let $C_u$ be a cluster in class 0. Then, there are $2^{n-1}$ disjoint virtual Hamiltonian cycles in the $DC(n)$ so each cycle passes exactly one cube-edge $(u, v)$ in the $C_u$ with $u = v \oplus 2^0$.

Let $VHC$ be a virtual Hamiltonian cycle in a $DC(n)$. We called a vertex $u$ is adjacent to $VHC$ if there exists a vertex lying on $VHC$ connecting to $u$. Thus, we obtain

**Lemma 5** For $n \geq 2$, let $VHCS = \{VHC_i \mid i = 0, 1, \ldots, 2^{n-1} - 1\}$ be a set of $2^{n-1}$ disjoint virtual Hamiltonian cycles established by algorithm dualcube$VHC$ in a $DC(n)$. Then, for any vertex $u$ in $VHC_i$, $0 \leq i \leq 2^{n-1} - 1$, $u$ is adjacent to $n - 1$ distinct virtual Hamiltonian cycles differing from $VHC_i$ in $VHCS$.

**Proof.** Let $0 \leq i \leq 2^{n-1} - 1$ and $u$ be a vertex in $VHC_i$. Since the cluster $C_u$ is isomorphic to an $n$-dimensional hypercube $Q_n$, there are $n$ cube-edges in $C_u$ incident to $u$. Let $(u, u_j)$ be a cube-edge in $C_u$ for $0 \leq j \leq n - 1$. Without loss of generality, we assume $(u, u_0)$ is a cube-edge lying on $VHC_i$. Suppose there are two vertices $u_k$ and $u_l$ lying on the same virtual Hamiltonian cycle $VHC_m$ in $VHCS$ for some $0 \leq k, l \leq n - 1$. Since only two neighboring vertices in each cluster are contained in $VHC_m$, $h(u_k, u_l) = 1$. Note, $h(u_k, u_l) = 2$ for $0 \leq k, l \leq n - 1$. A contradiction occurs. Therefore, $u$ is adjacent to $n - 1$ disjoint virtual Hamiltonian cycles in $VHCS$ except one containing $u$. \[\square\]

4 An adaptive diagnostic algorithm for dual-cubes

A set of vertices $U$ is said to be testable by a set $W$ if each vertex in $U$ is connected to a vertex in $W$. If all vertices in $W$ are fault-free, we know the correctness of $U$ by $W$. Moreover
$U$ is diagnosable by $W$ in parallel if for any pair of vertices $u$ and $v$ in $U$, they connect two distinct vertices in $W$. A vertex $u$ is unrecognizing if its status is unknown. An edge $(u, v)$ is called an unrecognized edge if $u$ and/or $v$ is unrecognized. Moreover, a cluster $C_u$ of $DC(n)$ is unrecognized if it contains an unrecognized edge.

**Lemma 6** For $n \geq 2$ and $k \leq n - 1$, let $U$ be a set of $k$ disjoint unrecognized edges in $Q_n$ and all vertices in $V(Q_n) - V(U)$ are fault-free. Then, there exists a set $W$ in $Q_n$ with $2k$ fault-free vertices so $V(U)$ is diagnosable by $W$.

**Proof.** We use induction on $n$ to prove the lemma. For $n = 2$ and $k \leq 1$, the lemma holds. Assume the lemma is true for $n \leq m$ and $k \leq m - 1$. Let $n = m + 1$ and $U$ be a set of $k$, $k \leq m$, disjoint unrecognized edges in $Q_{m+1}$ and all vertices in $V(Q_{m+1}) - V(U)$ are fault-free. Since $u = v \oplus 2^i$ if $(u, v)$ is an edge in $Q_{m+1}$, let $E_i = \{ (u, v) | u = v \oplus 2^i, 0 \leq i \leq m \}$. Since $k \leq m$, there exists an integer $d \leq m$ so $E_d \cap U = \emptyset$. We partition $Q_{m+1}$ along dimension $d$ into two $m$-dimensional subcubes, denoted by $Q^0_m$ and $Q^1_m$. The proof is divided into two cases: (1) $V(U) \subseteq V(Q^i_m)$ for some $i = 0, 1$ and (2) $V(U) \cap V(Q^i_m) \neq \emptyset$ for all $i = 0, 1$.

Suppose that case (1) holds. All unrecognized vertices are located in $V(Q^i_m)$ for some $i = 0, 1$. Let $W = \{ v | v = u \oplus 2^i \}$. Obviously, $|W| = 2k$, $W \subseteq V(Q^{i-1}_m)$, and any vertex in $W$ is fault-free. Suppose that case (2) holds. Let $V(U)$ be partitioned into two subsets $U_0$ and $U_1$ so $U_i = U \cap E(Q^i_m)$ for $i = 0, 1$, where $k_i = |U_i|$ and $k = k_0 + k_1$. By the induction hypothesis, there is a set of $2k_i$ fault-free vertices $W_i$ in $Q^i_m$ so $V(U_i)$ is diagnosable by $W_i$ for $i = 0, 1$. Obviously, $W_0 \cap W_1 = \emptyset$. Let $W = W_0 \cup W_1$. Thus, $|W| = 2k_0 + 2k_1 = 2k$ and $V(U)$ is diagnosable by $W$. \[\square\]

According to previous lemma, an algorithm can be established by recursive to generate a fault-free set $W$ in $Q_n$ to diagnose a given unrecognized set $U$ satisfying the condition in Lemma 6. Suppose that $DC(n)$ has at most $n + 1$ faulty nodes and given a syndrome $\sigma$. 11
Since a virtual Hamiltonian cycle contains $2^{n+2}$ vertices in $DC(n)$ and by Lemma 3, any $\sigma$-zero virtual Hamiltonian cycle is fault-free.

**Lemma 7** For $n \geq 2$, let $VHC$ be a virtual Hamiltonian cycle containing at most $m$, $m \leq n+1$, faulty nodes in a $DC(n)$ and $\sigma$ is a syndrome on $VHC$. Then, there are $O(m^2)$ unrecognized vertices in the $VHC$ if the $VHC$ is $\sigma$-nonzero.

**Proof.** Since the $VHC$ has $2^{n+2}$ vertices and contains at most $m$ faulty nodes, there are at most $2m$ 1-arrows in it. Suppose that the $VHC$ contains $k$ consecutive 1-arrow substrings. Therefore, for each 1-arrow string, we should be check the fault-freeness of the consecutive $m+1-k$ vertices starting from the head of the 1-arrow. Thus, at most $k(m+1-k)$ vertices are unrecognized and the rest vertices are fault-free. Hence the lemma holds. \hfill \Box

The above discussion is summarized as follows.

**Theorem 1** For $n \geq 1$, the $(n+1)$-connected dual-cube $DC(n)$ with at most $n+1$ faulty nodes is adaptively diagnosed with at most 3 rounds and $N + O(n^3)$ tests in the algorithm Ada-DualCubeTest.

## 5 Conclusions

This paper address problem diagnosis in computer networks and distributed computer systems. In adaptive diagnosis, you may choose what tests to be made based on the outcomes of previous tests. Furthermore, it is assumed that the fault-status of a processor does not change during the diagnosis. In this paper, we propose an adaptive diagnostic algorithm using 3 testing rounds for the $(n+1)$-connected dual cube with at most $n+1$ faulty nodes. Moreover, we showed the algorithm only need $N + O(n^3)$ tests to locating faulty components in a distributed system modeled by a $DC(n)$ in which at most $n+1$ components are faulty, where $N = 2^{2n+1}$ is the size of the system.
Algorithm 2 Ada-DualCubeTest($n$)

1: **Step 1:** Decomposed $DC(n)$ into $2^{n-1}$ disjoint virtual Hamiltonian cycles $VHC_i$ by algorithm dualCubeVHC, $0 \leq i \leq 2^{n-1}$.

2: **Step 2:** Perform the first series of tests along all edges of the ring $VHC_i$ in the clockwise direction.

3: **Step 3:**
4: $T = \emptyset$;
5: **if** (there are at least $n$ virtual Hamiltonian cycles containing 1-arrow) **then**
6: **for** each $VHC_i$ containing 1-arrow **do**
7: **if** there is a sequence $a \downarrow b \uparrow c \uparrow d \downarrow e$ in the syndrome of $VHC_i$ **then**
8: $T = T \cup (e, d)$;
9: **end if**
10: **if** there is a sequence $a \downarrow b \uparrow c \downarrow d$ in the syndrome of $VHC_i$ **then**
11: $T = T \cup (d, c)$;
12: **end if**
13: **if** there is a sequence $a \downarrow b \uparrow c \downarrow d \downarrow e$ in the syndrome of $VHC_i$ **then**
14: $T = T \cup (e, d)$;
15: **end if**
16: **if** there is a sequence $a \downarrow b \uparrow c \downarrow d$ in the syndrome of $VHC_i$ **then**
17: $T = T \cup (d, c)$;
18: **end if**
19: **end for**
20: **else**
21: **for** (each unrecognized cluster $C_u$) **do**
22: Let $U = \{(u, v) \mid (u, v)$ is an unrecognized edge in $VHC_i \cap C_u$ for $0 \leq i \leq 2^{n-1}\}$. Find a fault-free vertex set $W$ to diagnose $U$ based on Lemma 6;
23: $T = T \cup W$;
24: **end for**
25: **end if**
26: **end if**
27: **Step 4:** Perform tests in $T$;
References


